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TABLES OF RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES

R. Gene Brown

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PREPARED FOR:
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The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

This Memorandum describes some results of research concerned with the preparation of management reports from sample data. It contains tables of sample sizes that should facilitate estimating mean and aggregate values of certain quantities. To mention a few examples, these tables can be used to estimate the value of inventory stored in a particular location, estimate the value of obsolete inventory, estimate dollar value and reliability of an account or accounting statement by examination of bona fide transaction documents, and verify the ledger value of an inventory account.

Complete tables are not reproduced within this Memorandum because of the large number of pages involved. If it seems desirable later on, we may publish additional parts that appear to have a broad application. These tables are limited in their use by (1) statistical considerations relative to underlying assumptions about the shape of the parent population (i.e., normally distributed) and (2) the type of sampling application (i.e., estimating the mean or aggregate unit or dollar values of a specified population).

This study will be of particular interest to the Auditor General, USAF and to other Air Force personnel concerned with the application of sampling techniques.

The author is a consultant to The RAND Corporation and is an Assistant Professor at the Harvard Graduate School of Business Administration. The suggestions and help of Max Astrachan and Murray Geisler, both of The RAND Corporation, are gratefully acknowledged.

SUMMARY

This Memorandum presents examples of 16 tables of random sample sizes necessary to estimate mean and aggregate values. The tables provide a readily useful tool for determining an appropriate sample size once the person designing the plan makes certain quantitative statements.

The tables were developed as an aid in using sampling methods to estimate such financial characteristics of a total inventory as its aggregate dollar value or average value per line item stored. The computed tables permit the estimator to select the amount of precision and confidence desired in the estimate of total population characteristics.

Suppose, for example, we desire to estimate the value of an inventory containing 5000 line items, and the precision desired is to be within 5 per cent of the true value, with 95 per cent confidence. If the coefficient of variation is roughly 0.5, then the sample size, according to the tables (e.g., Table 6) in this Memorandum, would be 357 line items or about 7 percent of the items in the inventory. As might be expected, the sample size would vary considerably according to the confidence sought. For the same inventory, if a precision of 5 per cent with 90 per cent confidence were desired, the sample size would fall to 257, and for a 99.9 per cent confidence with the same precision it would increase to 891.

Some advantages of the tables are:

(1) A person does not have to be trained in statistics to implement an estimation sampling program of this type.

(2) Clerical errors and uncertainties in computing sample sizes are reduced.

(3) The comparative sample sizes (and hence, cost) for several different sampling plans can be compared.

In addition to the tables, a short discussion of the general method of estimation sampling in this area is included.

CONTENTS

I. INTRODUCTION

Since World War II, considerable progress in sampling theory and methodology has taken place. The successful use of acceptance sampling in statistical quality control encouraged individuals in other professions to consider applying statistical sampling to their own occupations. Frequently they found that transferring knowledge from one field to another was not as simple as it first appeared. In other applications, many sampling problems arose which simply did not fit into the decision framework of traditional industrial quality control.

For example, the accounting profession applied sampling techniques in editing data, controlling the quality of clerical activities, performing tests of transaction documents to estimate error rates, estimating inventory values, aging accounts, etc. To perform these tasks, new techniques had to be developed. Despite the availability of new methodology, successful applications have not been extensive.

This is the case principally because few non-statisticians have the training or the inclination to use unfamiliar and sometimes cumbersome statistical methods, while the statisticians have often been preoccupied with theoretical considerations. Even when the two parties occasionally get together, they find that lay and statistical jargon do not facilitate communication. An excellent example of this is the concept of confidence,* which to the statistician has a special and restricted definition that probably fits only a small category of the lay uses of that term.

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*For a description of the differences in meaning associated with common terms see Nehemiah Jordan, Decision Making Under Uncertainty and Problem Solving: A Gestalt Theoretical Viewpoint, The RAND Corporation, P-2156, December 1, 1960, Section II.

This interdisciplinary communication problem is amplified by the impatience of both parties in learning the other's vocabulary and conceptual framework. To bridge this gap and to provide tools which can be readily used in applications of sampling methods, there have been many recent sampling tables published.* These tables provide the layman a means of implementing probability sampling techniques without forcing him to become conversant with the complexities of statistical computations.

It is easy to become impatient with laymen who desire to use statistical methods, yet do not wish to understand the finer points of the underlying mathematics. However, this bitter pill must be swallowed. One does not need to know how to dismantle a dual carburetor in order to drive a modern automobile; similarly, the argument goes that it is not necessary to understand the refinements of statistical mathematics in order to use properly constructed sampling tables. The successful use of acceptance sampling tables such as the MIL-STD Series and the Dodge-Romig tables lends credence to this argument.

The tables illustrated in this study are the first extensive set prepared for use in estimating mean and aggregate values.

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*See Auditor General USAF, Tables of Probabilities for Use in Stop-or-Go Sampling, Government Printing Office, 1962; R. Gene Brown and Lawrence L. Vance, Sampling Tables for Estimating Error Rates or Other Proportions, Bureau of Business and Economic Research, University of California, 1961; and Murray A. Geisler, The Sizes of Simulation Samples Required to Compute Certain Inventory Characteristics with Stated Precision and Confidence, The RAND Corporation, RM-3242-PR, August, 1962.

II. ESTIMATION SAMPLING

The objective of estimation sampling is to make certain quantitative inferences about the characteristics of the universe* under study. It is possible to estimate proportion (rate) of error or mean and aggregate values. For example, an inventory of warehouse items could be taken using estimation sampling techniques. From the sample results, a quantitative estimate of the inventory's total dollar value could be made.

In a particular case of this type, if the estimation sample size derived were 357, then that number of inventory line items would be selected at random and their mean dollar value computed. This value would then be converted to the universe value by a proportion or ratio calculation. For example, if the total inventory value were desired, this mean value would be multiplied by 5000 if that was the number of line items in the universe.

In order to determine an appropriate sample size to perform tests like this, certain quantitative statements must be made: (1) the universe must be defined and delineated; (2), (3) the confidence and the precision desired in the estimate must be stated; and (4) the variability in the universe must be estimated. Since the universe is the mass from which the sample is to be selected, two of its characteristics should be determined in the early stages of the sampling process. These are the homogeneity and the size of the mass.

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*The universe is the mass of population whose characteristics are unknown, and therefore must be estimated through sampling.

The universe should be reasonably uniform throughout, both for content and processing (i.e., administrative review and recording flow). In the case study outlined in another paper,* this was extremely difficult to determine.

Items belonging to different classes or those having materially different characteristics must be segregated for special study. An example of this might be segregation and identification of non-serviceable items from the remaining inventory.

The universe definition includes the determination of its size, which can be found in various ways. For example, internal information is often available (item count, item serial number, and so on) which would give a reasonable estimate of the total universe number. If no such information is accessible, an estimate on the high side should be made. If the mass is not readily divisible into groups, as in the case of a flow of input transactions from a key punch operation or transceiver network, or if the universe is quite large, it is reasonable to assume the universe to be of infinite size. Such an assumption should not have any major effect on the results obtained with the sampling plan. In fact, it would be possible to assume an infinite universe size in all cases, but for smaller groups this is inefficient. Reference to the sampling tables included here will clarify this relationship.

Better information can often be obtained by separating the material under examination into two or more homogeneous groups. Estimates of each group's characteristics can be made on the basis of

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*R. Gene Brown, Inductive Accounting, The RAND Corporation, P-2671, September, 1962.

separate samples, and then combined for over-all analysis. This is the procedure of stratification which usually must be done when the universe has heterogeneous elements that can be subdivided into more homogeneous groups. The decision to stratify is often based on the information developed either from pre-planned tests, or immediately following the selection of the initial sample units when it becomes apparent that more than one group exists. Whether or not it is desirable to stratify the mass, the basic problems relative to definition and delineation of the universe remain those of determining homogeneity and size.

The estimator usually selects both precision and confidence levels judgmentally. Precision specifies the maximum desired difference between the sample estimate and the true, but unknown, universe mean value.* Confidence level measures the assurance desired that the interval calculated from the specified precision and the sample estimate will contain the actual universe mean value. For example, plus and minus $500 might be specified as the desired precision for estimating an inventory's mean value; 95 per cent might be the confidence "level". If the resulting sample size were 1,000, it would be expected that 95 per cent of a very large number of samples of size 1,000 drawn from this universe would provide an unbiased estimate of the true mean value within $500. Then it is reasonable to state that if, from a given sample, the estimated mean value of the universe were $15,000, the sample would provide 95 per cent confidence that the true universe mean value is included in the interval from $14,500 to $15,500.

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*If the precision is specified for the aggregate inventory, the precision desired for the mean value is obtained by dividing the former value by the number of items in the universe.

For the majority of the samples possible from this universe, the sample mean estimate will actually be much closer to the true value than $500. This is true because estimates of mean or aggregate values from a series of samples drawn from one universe will tend to be distributed normally about the actual unknown value, so that there is more sample clustering in the middle of the precision range than at its ends.

The degree of variability existing among the individual values within the universe is the final quantitative determination required. Variability is usually expressed in terms of the standard deviation. For the sampling tables in this Memorandum, we have to use the so-called coefficient of variation, which is the universe mean divided by the universe standard deviation. Since both the mean and the standard deviation are usually unknown, we have to estimate them. A procedure for doing this follows.*

We select randomly from the universe a pilot sample of about 30 to 50 line items, and compute its mean and standard deviation. With these data, we can then estimate the coefficient of variation. If the pilot mean is $138.46 and the pilot standard deviation is $25.50, the coefficient of variation to be used is 25.50/138.46 = 0.2.

In using the sample size tables, it will also be necessary to convert the absolute precision desired to relative terms. To do this we take the precision desired for the estimate of the mean value and divide it by the pilot sample mean. Thus, if the absolute precision

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*Much of the statistical development contained here has been drawn from L. L. Vance and J. Neter, Statistical Sampling for Auditors and Accountants, John Wiley and Sons, Inc., New York, 1956.

desired is \$10.00, and the pilot mean value is \$138.46, then the relative precision is 10.00/138.46 = 0.07 or ±7 per cent.

Once we draw the pilot sample and make the required calculations, we have the data necessary to use the tables in the Appendix. To recapitulate, we have to know: (1) the universe size, (2) the absolute precision desired in estimating the universe mean, (3) the confidence desired in the estimate, (4) the pilot sample mean, (5) the pilot sample standard deviation. With these data, we can use the procedure illustrated in Sec. III for obtaining the required sample size from the tables.

III. USING THE TABLES

For purposes of illustration, assume that we want to estimate the total dollar value of the ending inventory for a given accounting period. This estimate will be used as a check on the reliability of the value shown in the general ledger.

We took a small pilot sample to provide quantitative data concerning the universe characteristics. Following this, we made decisions concerning the other parameters required to select the sampling plan. Suppose the following data were then available:

Universe size	4,896
Pilot sample estimate of mean	\$ 430.19
Pilot estimate sample of standard deviation	\$ 212.65
Confidence level desired	95%
Absolute precision desired in estimating true mean	\$ 20.00

With the above information, a sample size can then be computed or can be obtained from tables like the ones in this Memorandum. In order to use the tables, the data must be arranged to correspond to the values required to enter the tables. This results in the following:

Universe size (rounded up)	5,000
Confidence level	95%
Relative precision (\$20/\$430.19)	±5% (rounded up)
Coefficient of variation (\$212.65/\$430.19)	0.5 (rounded up)

Referring to Table 6 in the Appendix, we find that a sample of 357 line items selected at random from the 4,896 item universe would satisfy this sampling plan.

It is interesting to note how the sample size would change when different levels of confidence and precision are specified for the same universe size and coefficient of variation. Referring again to the tables, a universe of 5000 items and a 0.5 coefficient of variation yields the following sample sizes for the confidence levels and precision stated:

Table 1

EFFECT OF VARYING PRECISION AND CONFIDENCE LEVEL ON SAMPLE SIZE

Precision (+ & -)	Confidence Level (%)	Appendix Table	Sample Size
0.050	90.0	2	257
0.050	95.0	6	357
0.050	99.0	10	586
0.050	99.9	14	891
0.010	95.0	6	3,289
0.020	95.0	6	1,623
0.040	95.0	6	536
0.100	95.0	6	95

Thus, comparative costs of different sampling plans can also be easily approximated by perusal of the tables.

Our 357 sample size example indicates the number of inventory items which have to be randomly selected and whose mean value has to be determined in order to estimate the aggregate universe value with desired precision and confidence. Suppose the mean value of the dollar amounts of the 357 items is $420.82. Multiplying this value by 4,896 yields an estimated total inventory value of $2,060,334. Compare this to the ending inventory value recorded in the general ledger (e.g., $2,142,418). Since the difference is less than 4%, the accounting records could reasonably be accepted as representing fairly the actual value of the physical stock on hand.

If there were serious disagreement between the sample estimate and the general ledger, several courses of action might follow, the most likely one being that a 100 per cent physical inventory would be taken.

IV. STATISTICAL CONSIDERATIONS

As indicated previously, the statistical basis for the sampling approach taken here is given in Vance and Neter.* We will trace through the derivation of the main formulae used in calculating the Appendix tables. Their results depend upon the underlying statistical assumption that the characteristic measured is normally distributed. Although this requirement is probably not satisfied in all cases, the fact that our interest usually lies in measuring the mean value of some characteristic suggests that, by the Central Limit Theorem,** the normality condition will be reasonably well satisfied for purposes of this application.

We make use of the following notation:

- N = population size;
- n = sample size;
- σ = universe standard deviation;
- s = sample estimate of universe standard deviation;
- μ = universe mean;
- x_i = characteristic of i-th (dollar value, number of units, etc.);
- $\bar{x}$ = sample estimate of population mean;
- v = coefficient of variation;
- d = absolute precision desired in estimating true mean;
- a = relative precision desired; and
- t_α = normal deviate corresponding to desired confidence level (thus, $t_\alpha = 1.96$ for confidence level of 95 percent).

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*Ibid., particularly Chapter 10.

**William Feller, An Introduction to Probability Theory and Its Application, John Wiley & Sons, Inc., 1959, Chapter 10.

The standard deviation of any population distribution can be defined as

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}} .$$

Since the true standard deviation of the universe is not known, it is usually estimated on the basis of a pilot sample of from 30 to 50 items. The pilot sample estimate of the population standard deviation is obtained from

$$s_p = \sqrt{\frac{N - 1}{N}} \sqrt{\frac{\sum_{i=1}^{n_p} (x_i - \bar{x}_p)^2}{n_p - 1}} ,$$

where s_p = standard deviation of the pilot sample, $\bar{x}_p$ = mean of the pilot sample, and n_p = pilot sample size. Once s_p is obtained, the standard deviation of the distribution of the sample mean for sample of size n can be estimated from the following formula:*

$$s_{\bar{x}} = \sqrt{\frac{N - n}{N - 1}} \frac{s_p}{\sqrt{n}} .$$

Solving the formula above for n gives

$$n = \frac{\frac{N}{N - 1} s_p^2}{s_{\bar{x}}^2 + \frac{s_p^2}{N - 1}} .$$

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*Vance and Neter, op cit, p. 193.

Or, since N is large and the absolute precision desired, d, can be set equal to $t_\alpha s_{\bar{x}}$ for confidence level α,

(1) $$n = \frac{s_p^2}{\left(\frac{d}{t_\alpha}\right)^2 + \frac{s_p^2}{N}} .$$

The formula can also be stated in relative terms, using the coefficient of variation. From above, we have:

$$n = \frac{s_p^2}{\left(\frac{d}{t_\alpha}\right)^2 + \frac{s_p^2}{N}}$$

$$= \frac{1}{\frac{1}{s_p^2}\left(\frac{d}{t_\alpha}\right)^2 + \frac{1}{N}}$$

$$= \frac{1}{\frac{\bar{x}_p^2}{s_p^2} \cdot \frac{1}{\bar{x}_p^2}\left(\frac{d}{t_\alpha}\right)^2 + \frac{1}{N}}$$

$$= \frac{1}{\left(\frac{d}{\bar{x}_p v t_\alpha}\right)^2 + \frac{1}{N}} , \quad \text{since } v = \frac{s_p}{\bar{x}_p} ,$$

$$= \frac{1}{\left(\frac{a}{v t_\alpha}\right)^2 + \frac{1}{N}} ,$$

where $$a = \frac{d}{\bar{x}_p} .$$

Finally, to facilitate computation, we can invert the formula to obtain:

(2) $$\frac{1}{n} = \left(\frac{a}{vt_\alpha}\right)^2 + \frac{1}{N} \quad .$$

We have used (1) and (2) to compute the sample sizes with the data presented earlier in this Memorandum. These data were:

Universe size	4,896
Pilot sample estimate of mean	$430
Pilot sample estimate of standard deviation	$213
Confidence desired	95 per cent
Absolute precision desired............	$20

From this information, we find that $a = 0.05$, $v = 0.5$, and $t_\alpha = 1.96$. Substituting these values in (1) gives $n = 369$, and in (2) gives 373. From Table 6, $n = 367$. The differences are due to rounding errors.

V. BREADTH OF THE TABLES

The tables illustrated in this Research Memorandum are extracts of complete ones which were programmed and run in the Data Services Division at Norton Air Force Base. The tables reproduced herein are incomplete as to all values computed and were selected merely to illustrate the ranges of some of the computations.

The actual range of values in the complete tables is as follows:

Table 2

RANGE OF VALUES FOR COMPUTED SAMPLE SIZE TABLES

Confidence Level	Coefficient of Variation		Universe Size		Relative Precision
80.0%	0.001	0.450	25	1,500	0.005
90.0%	0.002	0.500	50	1,750	0.010
95.0%	0.003	0.600	75	2,000	0.015
99.0%	0.004	0.700	100	2,250	0.020
99.9%	0.005	0.800	150	2,500	0.025
	0.006	0.900	200	3,000	0.030
	0.007	1.000	250	3,500	0.035
	0.008	1.100	300	4,000	0.040
	0.009	1.200	350	4,500	0.045
	0.010	1.300	400	5,000	0.050
	0.020	1.400	450	6,000	0.060
	0.030	1.500	500	7,000	0.070
	0.040	1.600	550	8,000	0.080
	0.050	1.800	600	9,000	0.090
	0.060	2.000	650	10,000	0.100
	0.070	2.500	700	15,000	0.150
	0.080	3.000	750	20,000	0.200
	0.090	3.500	800	25,000	0.250
	0.100	4.000	850	30,000	
	0.150	4.500	900	35,000	
	0.200	5.000	1,000	40,000	
	0.250	6.000	1,100	50,000	
	0.300	8.000	1,200	100,000	
	0.350	10.000	1,300	Infinite	
	0.400	20.000	1,400		

APPENDIX

Table 1

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES: COEFFICIENT OF VARIATION: 0.1 ; CONFIDENCE LEVEL: 90.0%

Universe Size	Relative Precision 0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	23	19	14	11	8	3	2	1
50	48	43	29	19	13	9	3	2	1
100	92	74	41	24	15	10	3	2	1
250	204	130	54	27	16	11	3	2	1
500	342	176	60	29	17	11	3	2	1
750	443	199	63	29	17	11	3	2	1
1,000	520	213	64	30	17	11	3	2	1
2,000	703	239	66	30	17	11	3	2	1
3,000	796	249	67	30	17	11	3	2	1
4,000	852	254	67	30	17	11	3	2	1
5,000	890	257	67	30	17	11	3	2	1
10,000	977	264	68	30	17	11	3	2	1
25,000	1,038	268	68	31	17	11	3	2	1
50,000	1,060	270	68	31	17	11	3	2	1
100,000	1,071	270	68	31	17	11	3	2	1
Infinite	1,082	271	68	31	17	11	3	2	1

Table 2

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES: COEFFICIENT OF VARIATION: 0.5 ; CONFIDENCE LEVEL: 90.0%

Universe Size	Relative Precision 0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	24	23	19	14	8
50	50	50	49	47	45	44	29	19	9
100	100	99	95	89	81	74	41	24	10
250	248	242	218	188	158	130	54	27	11
500	491	466	386	301	230	176	60	29	11
750	730	676	520	376	271	199	63	29	11
1,000	965	872	629	430	298	213	64	30	11
2,000	1,863	1,544	917	547	349	239	66	30	11
3,000	2,701	2,079	1,082	601	371	249	67	30	11
4,000	3,485	2,514	1,189	633	383	254	67	30	11
5,000	4,221	2,875	1,264	654	390	257	67	30	11
10,000	7,302	4,035	1,447	700	406	264	68	30	11
25,000	12,994	5,324	1,584	730	416	268	68	31	11
50,000	17,556	5,958	1,636	741	420	270	68	31	11
100,000	21,295	6,336	1,663	746	421	270	68	31	11
Infinite	26,343	6,719	1,689	751	423	271	68	31	11

Table 3

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES: COEFFICIENT OF VARIATION: 1.0 ; CONFIDENCE LEVEL: 90.0%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	25	23	21	16
50	50	50	50	50	49	48	43	36	24
100	100	100	99	97	95	92	74	55	31
250	250	248	242	231	218	204	130	82	37
500	498	491	466	429	386	342	176	97	40
750	745	730	676	601	520	443	199	104	41
1,000	991	965	872	751	629	520	213	108	42
2,000	1,964	1,863	1,544	1,201	917	703	239	114	43
3,000	2,920	2,701	2,079	1,502	1,082	796	249	116	43
4,000	3,858	3,485	2,514	1,717	1,189	852	254	117	43
5,000	4,780	4,221	2,875	1,878	1,264	890	257	118	43
10,000	9,155	7,302	4,035	2,312	1,447	977	264	119	44
25,000	20,309	12,994	5,324	2,684	1,584	1,038	268	120	44
50,000	34,200	17,556	5,958	2,836	1,636	1,060	270	120	44
100,000	51,975	21,295	6,336	2,919	1,663	1,071	270	121	44
Infinite	97,654	26,343	6,719	2,998	1,689	1,082	271	121	44

Table 4

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES: COEFFICIENT OF VARIATION: 2.5 ; CONFIDENCE LEVEL: 90.0%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	25	25	25	23
50	50	50	50	50	50	50	49	49	43
100	100	100	100	100	100	99	95	89	74
250	250	250	249	247	245	242	218	188	130
500	500	499	495	488	478	466	386	301	176
750	750	747	737	722	701	676	520	376	199
1,000	999	995	977	950	914	872	629	430	213
2,000	1,995	1,977	1,910	1,808	1,682	1,544	917	547	239
3,000	2,987	2,948	2,802	2,587	2,337	2,079	1,082	601	249
4,000	3,977	3,908	3,655	3,298	2,902	2,514	1,189	633	254
5,000	4,964	4,857	4,472	3,950	3,395	2,875	1,264	654	257
10,000	9,855	9,442	8,088	6,527	5,139	4,035	1,447	700	264
25,000	24,103	21,780	15,710	10,727	7,429	5,324	1,584	730	268
50,000	46,559	38,590	22,907	13,657	8,725	5,958	1,636	741	270
100,000	87,120	62,839	29,714	15,817	9,559	6,336	1,663	746	270
Infinite	403,480	144,639	40,560	18,443	10,459	6,719	1,689	751	271

Table 5

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES:
COEFFICIENT OF VARIATION: 0.1 ; CONFIDENCE LEVEL: 95.0%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	24	20	16	13	10	4	2	1
50	49	45	33	24	17	12	4	2	1
100	94	80	49	30	20	14	4	2	1
250	216	152	70	37	22	15	4	2	1
500	378	218	81	40	23	15	4	2	1
750	504	255	86	41	24	16	4	2	1
1,000	606	278	88	41	24	16	4	2	1
2,000	869	323	92	42	24	16	4	2	1
3,000	1,017	341	94	43	24	16	4	2	1
4,000	1,111	351	94	43	24	16	4	2	1
5,000	1,176	357	95	43	24	16	4	2	1
10,000	1,332	370	96	43	24	16	4	2	1
25,000	1,448	379	96	43	24	16	4	2	1
50,000	1,491	382	96	43	24	16	4	2	1
100,000	1,514	383	96	43	25	16	4	2	1
Infinite	1,535	384	97	43	25	16	4	2	1

Table 6

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES:
COEFFICIENT OF VARIATION: 0.5 ; CONFIDENCE LEVEL: 95.0%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	24	20	16	10
50	50	50	49	48	47	45	33	24	12
100	100	99	97	92	121	80	49	30	14
250	249	244	227	203	177	152	70	37	15
500	494	476	414	341	273	218	81	40	15
750	736	696	572	441	334	255	86	41	16
1,000	975	906	706	517	376	278	88	41	16
2,000	1,902	1,656	1,092	696	462	323	92	42	16
3,000	2,783	2,286	1,334	788	501	341	94	43	16
4,000	3,623	2,824	1,501	843	522	351	94	43	16
5,000	4,425	3,289	1,623	880	536	357	95	43	16
10,000	7,935	4,899	1,937	965	567	370	96	43	16
25,000	15,145	6,939	2,191	1,024	587	379	96	43	16
50,000	21,925	8,057	2,291	1,045	594	382	96	43	16
100,000	27,754	8,763	2,345	1,056	597	383	96	43	16
Infinite	36,994	9,513	2,396	1,066	600	384	97	43	16

Table 7

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES:
COEFFICIENT OF VARIATION: 1.0 ; CONFIDENCE LEVEL: 95.0%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	25	24	22	18
50	50	50	50	50	49	49	45	39	28
100	100	100	99	98	97	94	80	64	39
250	250	249	244	237	227	216	152	102	50
500	499	494	476	448	414	378	218	128	55
750	747	736	696	638	572	504	255	140	57
1,000	994	975	906	811	706	606	278	146	58
2,000	1,975	1,902	1,656	1,362	1,092	869	323	158	60
3,000	2,943	2,783	2,286	1,762	1,334	1,017	341	162	61
4,000	3,899	3,623	2,824	2,065	1,501	1,111	351	164	61
5,000	4,843	4,425	3,289	2,303	1,623	1,176	357	166	61
10,000	9,389	7,935	4,899	2,992	1,937	1,332	370	168	62
25,000	21,502	15,145	6,939	3,646	2,191	1,448	380	170	62
50,000	37,725	21,725	8,057	3,933	2,291	1,491	382	171	62
100,000	60,577	27,754	8,763	4,094	2,345	1,514	383	171	62
Infinite	133,193	36,994	9,513	4,251	2,396	1,535	384	171	62

Table 8

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES:
COEFFICIENT OF VARIATION: 2.5 ; CONFIDENCE LEVEL: 95.0%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	25	25	25	24
50	50	50	50	50	50	50	49	48	45
100	100	100	100	100	100	99	97	92	80
250	250	250	249	248	246	244	227	203	152
500	500	499	496	491	484	476	414	341	218
750	750	748	741	730	715	696	572	441	255
1,000	999	996	984	964	938	906	706	517	278
2,000	1,996	1,984	1,936	1,861	1,765	1,656	1,092	696	323
3,000	2,991	2,963	2,858	2,697	2,501	2,286	1,334	788	341
4,000	3,984	3,935	3,751	3,479	3,159	2,824	1,501	843	351
5,000	4,975	4,898	4,616	4,211	3,751	3,289	1,623	880	357
10,000	9,897	9,601	8,572	4,274	6,001	4,899	1,937	965	370
25,000	24,366	22,643	17,650	12,906	9,378	6,939	2,191	1,024	379
50,000	47,526	41,383	27,278	17,396	11,542	8,057	2,291	1,045	382
100,000	90,570	70,597	37,509	21,059	13,048	8,763	2,345	1,056	383
Infinite	489,892	193,608	56,025	25,984	14,784	9,513	2,396	1,066	384

Table 9

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES: COEFFICIENT OF VARIATION: 0.1 ; CONFIDENCE LEVEL: 99.0%

Universe Size	Relative Precision 0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	22	19	16	13	6	3	2
50	50	47	39	30	23	18	6	3	2
100	97	87	63	43	30	21	7	3	2
250	229	182	100	57	36	24	7	3	2
500	421	286	125	65	39	26	7	3	2
750	585	353	136	68	40	26	7	3	2
1,000	727	399	143	69	40	26	7	3	2
2,000	1,141	499	154	72	41	27	7	3	2
3,000	1,409	544	158	72	41	27	7	3	2
4,000	1,596	570	160	73	42	27	7	3	2
5,000	1,734	586	161	73	42	27	7	3	2
10,000	2,098	623	164	74	42	27	7	3	2
25,000	2,400	647	165	74	42	27	7	3	2
50,000	2,521	655	166	74	42	27	7	3	2
100,000	2,586	660	166	74	42	27	7	3	2
Infinite	2,647	664	166	74	42	27	7	3	2

Table 10

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES: COEFFICIENT OF VARIATION: 0.5 ; CONFIDENCE LEVEL: 99.0%

Universe Size	Relative Precision 0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	25	22	19	13
50	50	50	50	49	48	47	39	30	18
100	100	100	98	95	92	87	63	43	21
250	250	247	236	221	202	182	100	57	25
500	499	486	447	394	338	286	125	65	26
750	742	718	636	534	436	353	136	68	26
1,000	986	944	806	649	510	399	143	69	26
2,000	1,942	1,785	1,350	960	683	499	154	72	27
3,000	2,871	2,541	1,741	1,142	771	544	158	72	27
4,000	3,773	3,223	2,037	1,262	824	570	160	73	27
5,000	4,650	3,842	2,267	1,347	859	586	161	73	27
10,000	8,691	6,239	2,932	1,557	940	623	164	74	27
25,000	18,159	9,972	3,557	1,717	996	647	165	74	27
50,000	28,513	12,456	3,830	1,778	1,016	655	166	74	27
100,000	39,886	14,228	3,982	1,810	1,027	660	166	74	27
Infinite	62,221	16,317	4,130	1,840	1,036	664	166	74	27

Table 11

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES: COEFFICIENT OF VARIATION: 1.0 ; CONFIDENCE LEVEL: 99.0%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	25	25	24	21
50	50	50	50	50	50	50	47	43	34
100	100	100	100	99	98	97	87	75	52
250	250	250	247	242	236	229	182	136	75
500	500	497	486	469	447	421	286	186	88
750	748	742	718	681	636	585	353	212	93
1,000	997	986	944	881	806	727	399	228	96
2,000	1,986	1,942	1,785	1,574	1,350	1,141	499	257	101
3,000	2,967	2,871	2,541	2,123	1,741	1,409	544	269	103
4,000	3,941	3,773	3,223	2,594	2,037	1,596	570	275	104
5,000	4,908	4,650	3,842	2,980	2,267	1,734	586	279	104
10,000	9,637	8,691	6,239	4,244	2,932	2,098	623	287	106
25,000	22,848	18,159	9,972	5,694	3,557	2,400	647	292	106
50,000	42,074	28,513	12,456	6,425	3,830	2,521	655	294	106
100,000	72,633	39,886	14,228	6,866	3,982	2,586	660	295	107
Infinite	209,734	62,221	16,317	7,319	4,130	2,647	664	295	107

Table 12

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES: COEFFICIENT OF VARIATION: 2.5 ; CONFIDENCE LEVEL: 99.0%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	25	25	25	25
50	50	50	50	50	50	50	50	49	47
100	100	100	100	100	100	100	98	73	87
250	250	250	250	249	248	247	236	221	182
500	500	500	498	495	491	488	447	394	286
750	750	749	745	738	729	718	636	534	353
1,000	1,000	998	991	979	963	944	806	649	399
2,000	1,998	1,991	1,963	1,917	1,857	1,785	1,350	960	499
3,000	2,995	2,979	2,916	2,817	2,689	2,541	1,741	1,142	544
4,000	3,991	3,962	3,852	3,681	3,466	3,223	2,037	1,262	570
5,000	4,995	4,941	4,770	4,511	4,192	3,842	2,267	1,347	586
10,000	9,941	9,765	9,121	8,217	7,216	6,239	2,932	1,557	623
25,000	24,629	23,579	20,143	16,207	12,726	9,972	3,557	1,717	647
50,000	48,537	44,620	33,732	23,979	17,070	12,456	3,830	1,778	655
100,000	94,315	80,571	50,9[illegible]2	31,543	20,583	14,228	3,982	1,810	660
Infinite	623,881	293,128	93,[illegible]	44,047	25,263	16,317	4,130	1,840	664

Table 13

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES:
COEFFICIENT OF VARIATION: 0.1 ; CONFIDENCE LEVEL: 99.9%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	23	21	19	16	8	5	2
50	50	48	43	36	29	24	9	5	2
100	98	92	74	55	41	31	10	5	2
250	237	204	130	82	54	37	11	5	2
500	449	343	176	97	60	40	11	5	2
750	640	444	199	104	63	41	11	5	2
1,000	813	520	214	108	64	42	11	5	2
2,000	1,367	703	239	114	66	43	11	5	2
3,000	1,773	796	249	116	67	43	11	5	2
4,000	2,080	853	254	117	67	43	11	5	2
5,000	2,321	891	257	118	67	43	11	5	2
10,000	3,023	977	264	119	68	44	11	5	2
25,000	3,692	1,038	268	120	68	44	11	5	2
50,000	3,986	1,060	270	121	68	44	11	5	2
100,000	4,152	1,072	270	121	68	44	11	5	2
Infinite	4,313	1,082	271	121	68	44	11	5	2

Table 14

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES:
COEFFICIENT OF VARIATION: 0.5 ; CONFIDENCE LEVEL: 99.9%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	25	23	21	16
50	50	50	50	50	49	48	43	36	24
100	100	100	99	97	95	92	74	55	31
250	250	248	242	231	218	204	130	82	37
500	498	491	466	429	386	343	176	97	40
750	745	130	676	601	520	444	199	104	41
1,000	991	965	872	751	629	520	214	108	42
2,000	1,964	1,863	1,544	1,202	917	703	239	114	43
3,000	2,920	2,701	2,079	1,502	1,082	796	249	116	43
4,000	3,858	3,486	2,515	1,717	1,189	853	254	117	43
5,000	4,780	4,221	2,876	1,878	1,265	891	257	118	43
10,000	9,155	7,303	4,036	2,313	1,448	977	264	119	44
25,000	20,311	12,997	5,326	2,685	1,585	1,038	268	120	44
50,000	34,205	17,562	5,961	2,838	1,637	1,060	270	121	44
100,000	51,987	21,303	6,339	2,920	1,664	1,072	270	121	44
Infinite	97,698	26,356	6,722	2,999	1,689	1,082	271	121	44

Table 15

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES: COEFFICIENT OF VARIATION: 1.0 ; CONFIDENCE LEVEL: 99.9%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	25	25	24	22
50	50	50	50	50	50	50	48	46	39
100	100	100	100	100	99	98	92	83	64
250	250	250	248	245	242	237	235	165	103
500	500	498	491	481	466	449	343	246	129
750	749	745	730	706	676	640	444	294	141
1,000	998	991	965	924	872	813	520	325	148
2,000	1,991	1,964	1,863	1,715	1,544	1,369	703	388	160
3,000	2,980	2,920	2,701	2,402	2,079	1,773	796	415	164
4,000	3,964	3,858	3,486	3,002	2,515	2,080	853	430	167
5,000	4,943	4,780	4,221	3,533	2,876	2,321	891	439	168
10,000	9,775	9,155	7,303	5,461	4,036	3,023	977	460	171
25,000	23,636	20,311	12,997	8,123	5,326	3,692	1,038	473	173
50,000	44,826	34,205	17,562	9,698	5,961	3,986	1,060	477	173
100,000	81,242	51,987	21,303	10,739	6,339	4,152	1,072	479	173
Infinite	302,214	97,698	26,356	11,888	6,722	4,313	1,082	481	174

Table 16

RANDOM SAMPLE SIZES NECESSARY TO ESTIMATE MEAN AND AGGREGATE VALUES: COEFFICIENT OF VARIATION: 2.5 ; CONFIDENCE LEVEL: 99.9%

Universe Size	Relative Precision								
	0.005	0.010	0.020	0.030	0.040	0.050	0.100	0.150	0.250
25	25	25	25	25	25	25	25	25	25
50	50	50	50	50	50	50	50	50	48
100	100	100	100	100	100	100	99	97	92
250	250	250	250	250	249	248	242	231	235
500	500	500	499	497	495	491	466	429	343
750	750	750	747	743	737	730	676	601	444
1,000	1,000	999	995	987	977	965	872	751	520
2,000	1,999	1,995	1,977	1,949	1,910	1,863	1,544	1,202	703
3,000	2,997	2,987	2,948	2,885	2,802	2,701	2,079	1,502	796
4,000	3,995	3,977	3,908	3.798	3,655	3,486	2,515	1,717	853
5,000	4,991	4,964	4,857	4,689	4,492	4,221	2,876	1,878	891
10,000	9,964	9,855	9,442	8,828	8,088	7,303	4,036	2,313	977
25,000	24,772	24,110	21,782	18,762	15,713	12,997	5,326	2,685	1,038
50,000	49,094	46,560	38,594	30,031	22,913	17,562	5,961	2,838	1,060
100,000	96,438	87,126	62,851	42,920	29,724	21,303	6,339	2,920	1,072
Infinite	730,237	403,601	144,701	69,934	40,579	26,356	6,722	2,999	1,082